Inertial range scaling, Kármán-Howarth theorem, and intermittency for forced and decaying Lagrangian averaged MHD equations in two dimensions

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Summary

The Kármán-Howarth theorem was extended to the Lagrangian averaged magnetohydrodynamic (LAMHD-alpha) equations. The scaling laws resulting as a corollary of this theorem were studied in numerical simulations, as well as the scaling of the longitudinal structure function exponents indicative of intermittency. Numerical simulations were presented both for freely decaying and for forced two-dimensional MHD turbulence, solving directly the MHD equations, and employing LAMHD-alpha equations at 1/2 and 1/4 resolution. Linear scaling of the third-order structure function with length was observed. Remarkably, the LAMHD-alpha equations also captured the anomalous scaling of longitudinal structure function exponents up to order 8.

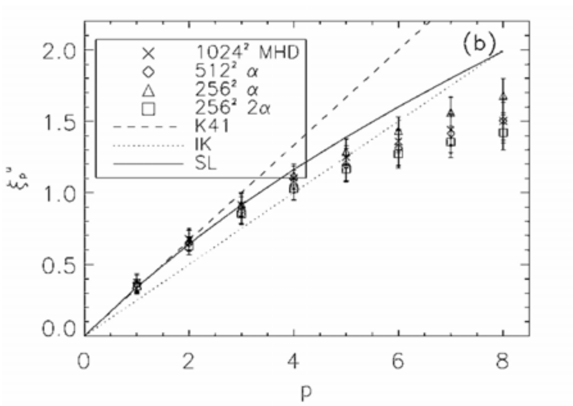

Figure 1. The scaling of structure function exponent versus order *p* computed over 189 turnover times shows that the LAMHD-alpha model preserves intermittency up to order 8. The labels are explained in the inset. The dashed line indicates K41 scaling, the dotted line indicates IK scaling, and the solid line is the prediction using a modified She-Lévêque formula [2]. In every case the intermittency indicated by this scaling is preserved very well by LAMHD-alpha.

Sufficient resolution for computing high Reynolds number flows as encountered in many DOE Laboratory missions is today well beyond technological limits. Closures such as the Lagrangian-averaged Navier-Stokes-alpha (LANS-alpha) model can reduce the computational burden by reducing the resolution requirements.

However, to be used as a model of either hydrodynamic or MHD turbulence, or for applications in astrophysics and geophysics, detailed quantification of the ability of the LANS-alpha, or LAMHD-alpha equations to capture key features of turbulent flows is required. The LANS-alpha and LAMHD-alpha equations have been tested against direct numerical simulations in a variety of problems, see [1,2] and references therein. Most of these tests compared the time evolution of ideal invariants for forced and free decaying turbulence, as well as the

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evolution of energy spectra and other statistical comparisons such as alignment.

Researchers at Los Alamos and their colleagues at NCAR have recently applied a much more stringent test to these models. ***Intermittency*** is a well-known feature of turbulent flows, associated with the existence of strong events localized both in space and time. Intermittency can trigger large-scale events, affect the transport coefficients, or give rise to corrections in the turbulent scaling. As a result, whether a model can capture the statistics of intermittent events is of utmost importance in high Reynolds number flows.

The study of intermittency also requires computation of high order statistics, thereby extending previous comparisons between DNS and alpha models. A generalization of the Kármán-Howarth theorem (KH-alpha) was proven for the LANS-alpha equations, as discussed in [1]. As a corollary of this theorem, Kolmogorov's K41 four-fifths law and –5/3 energy spectrum can be derived for the LANS-alpha equations at scales larger than alpha. The KH-alpha theorem was extended to the LAMHD-alpha case in [2], thereby proving for scales larger than alpha that the LAMHD-alpha equations also satisfy the scaling laws for MHD turbulence.

This is an important result, since MHD turbulence involves two coupled fields, the velocity and magnetic fields, and it can display different power laws in the inertial range according to the regime of interest. While Large Eddy Simulations (LES) often impose a particular regime and power law, the LAMHD-alpha equations are found to satisfy the proper scaling for solutions of the MHD equations without any hypothesis about the scaling in the inertial range.

Turbulence closures are never unique. The present case may owe its success not only to its particular form, but to its fundamental properties of: (1) preserving physical avenues of nonlinear energy exchange and (2) allowing correct vortex stretching. These two properties derive from its origin via a Lagrangian-averaged Hamilton's principle as described in [1]. The derivation also identifies the appropriate dissipation for proper energy decay, which for MHD involves an enhanced resistivity, but not an enhanced viscosity. Together, the Navier-Stokes viscosity and the enhanced resistivity produce regularization, e.g., existence and uniqueness of strong solutions and their global attractor of finite fractal dimension.

In turn, these choices of viscosity and resistivity allowed the intermittency found in [2], which might have otherwise been suppressed. Contrary to fluids, in two dimensions MHD turbulence displays a forward cascade of energy, as well as intermittency. The LAMHD-alpha equations reproduce proper intermittency features of turbulent flows and thus we postulate that these results will carry over to the three-dimensional case. Consequently, these results could be also of relevance to the 3D modeling of fluids. However, this matter was beyond the scope of [2].

References:

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