New Discretization Methodology on Generalized Polyhedral Meshes

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Summary

The growing number of complex engineering simulations shows advantage of using polyhedral meshes. Many of the existing numerical methods cannot be extended to polyhedral meshes, especially to meshes with cells having strongly curved faces. For a diffusion problem, we developed a new discretization methodology that overcomes this problem.

The polyhedral meshes are now used in complex engineering simulations and result in more accurate predictions than the tetrahedral meshes with a comparable number of cells [2]. Unfortunately, many of the existing numerical methods cannot be extended to polyhedral meshes, especially to generalized polyhedral meshes which have cells with strongly curved (nonplanar) faces.

In article [1], we considered a diffusion problem, which appears in computational fluid dynamics, heat conduction, radiation transport, etc., and developed a new discretization methodology that has no analogs in literature. The methodology follows the general principle of the mimetic finite difference (MFD) method to mimic the essential underlying properties of the continuum differential operators such as the conservation laws, solution symmetries, and the fundamental identities and theorems of vector and tensor calculus. For the diffusion problem, the MFD method mimics the Gauss divergence theorem, the symmetry between the gradient and divergence operators, and the null spaces of these operators.

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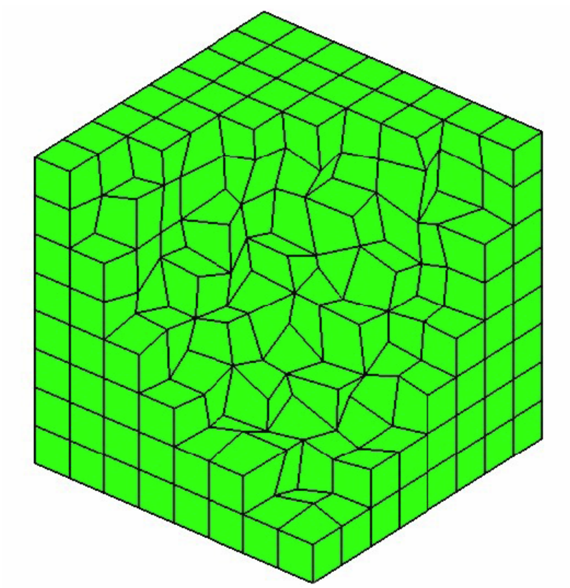
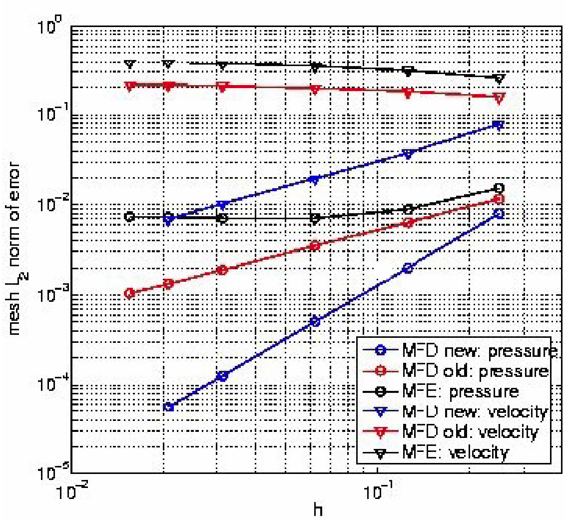
The new methodology improves drastically the capabilities of the existing discretization methods (see figures). It is computationally cheap and produces convergent symmetric and locally conservative discretization schemes.

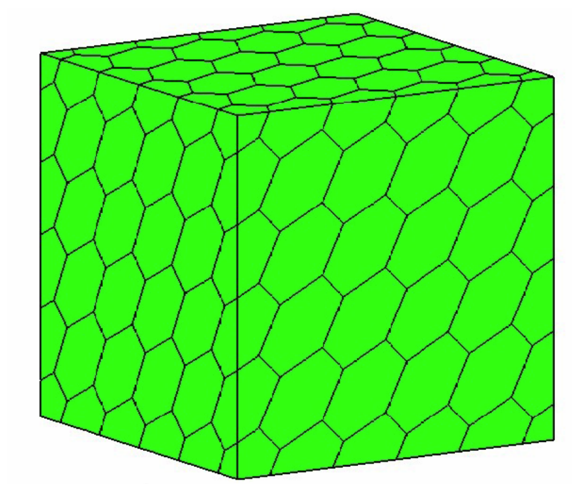


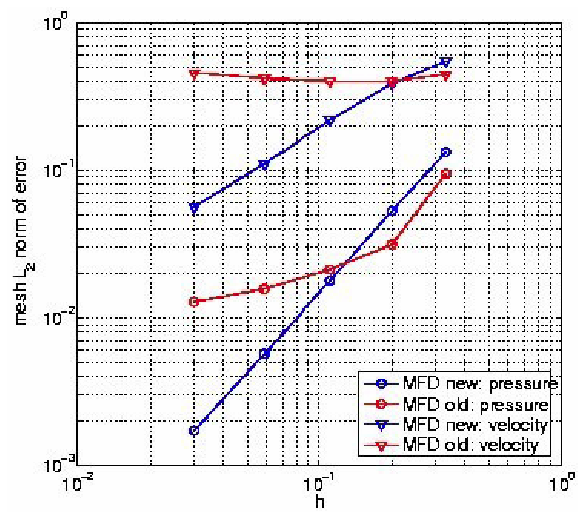


The top picture shows the interior of a logically cubic mesh with randomly perturbed points. The bottom picture shows the optimal convergence rates for the new MFD method (blue), and the lack of convergence for the mixed finite element (black) and the old MFD (red) methods.

[1] Brezzi, F., K. Lipnikov, M. Shashkov, *Math. Mod. Meth. Appl. Sci.* (2005), to appear.

[2] Peric, M., S. Ferguson, “The advantage of polyhedral meshes,” www.cdadapco.com/news/24/TetsvPoly.htm.





The top picture shows a generalized polyhedral mesh where the mixed finite element method cannot be used. Note that 68% of interior mesh faces are nonplanar. The bottom picture shows *optimal convergence rates for the new MFD method (blue) and the lack of convergence for the old MFD method (red).*

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