Advanced Scientific Computing Research
FY 2006 Accomplishment

Scalable Multilevel Preconditioners

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Summary

Our goal is to enable new simulation capabilities by extending the applicability of scalable algebraic multigrid solvers. This includes basic algorithm research and direct interaction with applications. FY2006 accomplishments center on smoothed aggregation for fluid flow and structural dynamics.

We recently developed a new smoothed aggregation algebraic multigrid method for non-symmetric systems. To the best of our knowledge, this is the first significant generalization of smoothed aggregation to non-symmetric systems. The new algorithm is significantly more robust and faster than *standard* smoothed aggregation over a variety of non-symmetric problems.

Figure 1 displays convergence histories on a realistic transonic 3D fluid flow calculation within a Sandia simulation. The plot considers the 1st linear system arising from Newton's method. The smoother is ILU(0). EMIN and EMINB (block variant) are the new methods while NSA and NSR are existing smoothed aggregation variants. As cost-per-iteration is similar between these schemes, this represents a *3x* to *4½x* improvement over previous methods. *Standard* smoothed aggregation does not converge and so is not shown.

The unique algorithm features include *restrictor smoothing* and a new way to define damping parameters needed to construct grid transfers. To develop the method, a Schur complement form of smoothed aggregation is considered in conjunction with local energy minimization based on the Frobenius norm. This leads to easily computable quantities that do not rely on symmetry. This technique has been combined with our recently developed block preconditioners for incompressible flow and exhibits mesh independent convergence over a range of Reynolds numbers.

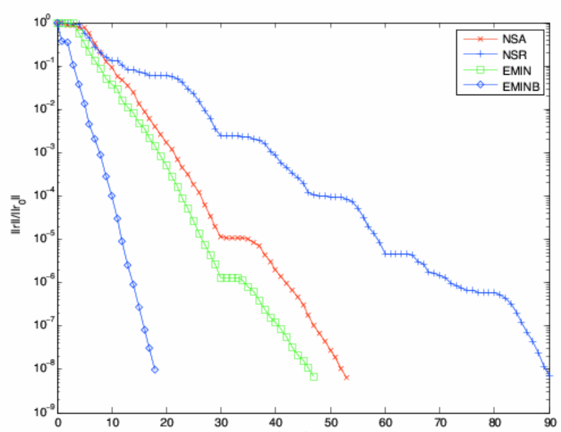

Figure 1

We have also made considerably progress in developing techniques for nonconforming discretizations (often used in structural dynamics at Sandia). The goal of this work centers on improving solution quality and reducing solution times to the underlying linear systems. A parallel package named

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Moertel has been developed based on dual and standard mortar methods. The package does not rely on a specific type of partial differential equation and so it can be used in mesh tying, contact, and surface coupled multi-physics. The mortar methods improve solution quality in the vicinity of nonconforming interfaces by an order of magnitude compared to simplistic node-to-segment approaches. Recently, capabilities to couple low order planar and warped interfaces in three dimensions were studied. A mortar algebraic multigrid method is under development and shows promise on the resulting constraint systems of equations. **Moertel** also serves as an *interface language* to study other nonconforming methods that we are investigating. **Moertel** will be released in September. http://software.sandia.gov/trilinos/packages/moertel/index.html

Finally, we are developing a new smoothed aggregation technique for anisotropic phenomena caused by mesh stretching or material properties. Due to poor smoothing in some directions, this phenomena leads to small aggregates within smoothed aggregation. Unfortunately, the interpolant (created by *prolongator smoothing*) contains too many nonzeros to be used. While fixes have been proposed, the most practical are not applicable to PDE systems nor to adaptive smoothed aggregation. Our new technique essentially uses a standard prolongator smoother to produce an initial grid transfer. This is then modified to significantly reduce the number of nonzeros while capturing zero-energy modes in the range of the prolongator. The advantage of this new approach is that it can be applied to PDE systems (e.g. elasticity) with an acceptable cost per iteration. The table below illustrates this for a 3D beam on a stretched mesh where iterations are given. BSSA corresponds to the new method. Both methods have a reasonable cost per iteration and ASA is our best attempt at applying traditional ideas to this problem.

# equations	ASA	BSSA
2187	42	14
10125	39	14
20577	55	12

Future work continues to be driven by specific applications. We are researching algebraic multigrid for nonaligned anisotropic behavior and considering new techniques to automatically detect anisotropy based on compatible relaxation. We are also combining this with new algorithm ideas for electro-magnetics.

[1] *Elman, Howle, Shadid, Shuttleworth & Tuminaro.* ***Block Preconditioners Based on Approximate Commutators***, *SISC*, Vol 27, 1651-1668, 2006.

[2] *Chow, Falgout, Hu, Tuminaro &Meier Yang.* ***A Survey of Parallelization Techniques for Multigrid Solvers***, *Frontiers of Parallel Processing For Scientific Computing, SIAM book series*'06.

[3] *Sala, Lin, Tuminaro, Shadid.* ***Algebraic Preconditioners for Non-symmetric PDEs on Stretched Grids***, *proceedings of 16th Domain Decomp, 2005.*

[4] *Sala, Tuminaro.* ***A New Petrov-Galerkin Smoothed Aggregation Preconditioner for Nonsymmetric Linear Systems***, *submitted to SISC, 2006.*

[5] *Elman, Howle, Silvester, Tuminaro.* ***Least Squares Preconditioners For Stabilized Discretizations Of The Navier-Stokes Equations***, *submitted to SISC, 2006.*

[6] *Gee, Tuminaro.* ***Nonlinear algebraic multigrid for constrained solid mechanics problems using Trilinos***, *Sandia Tech Report, 2006.*

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