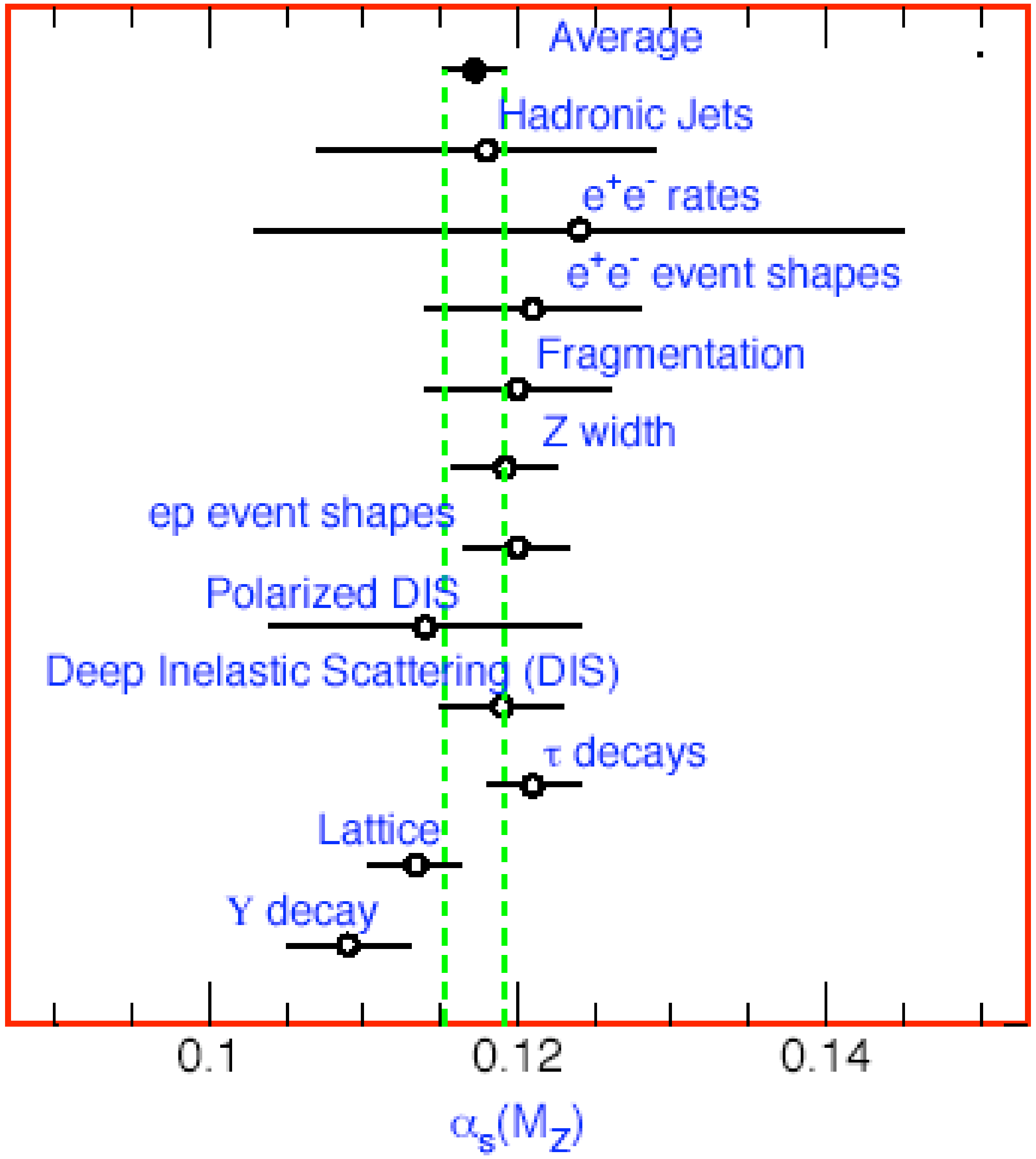
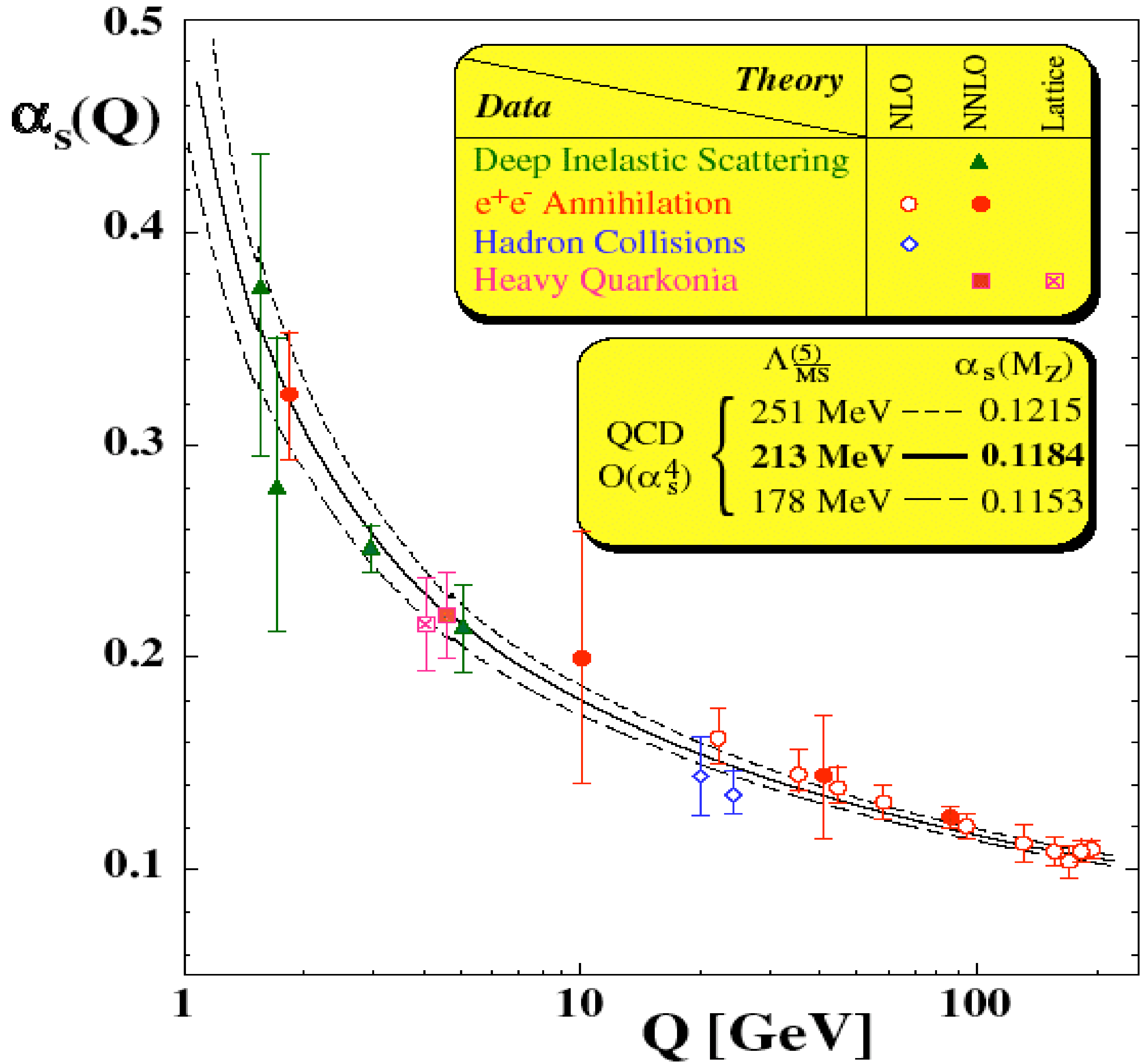
The Significance of Lattice Gauge Theory

Some Achievements, Some Anticipations

The Strong Interaction Coupling: Value, Running, Unification

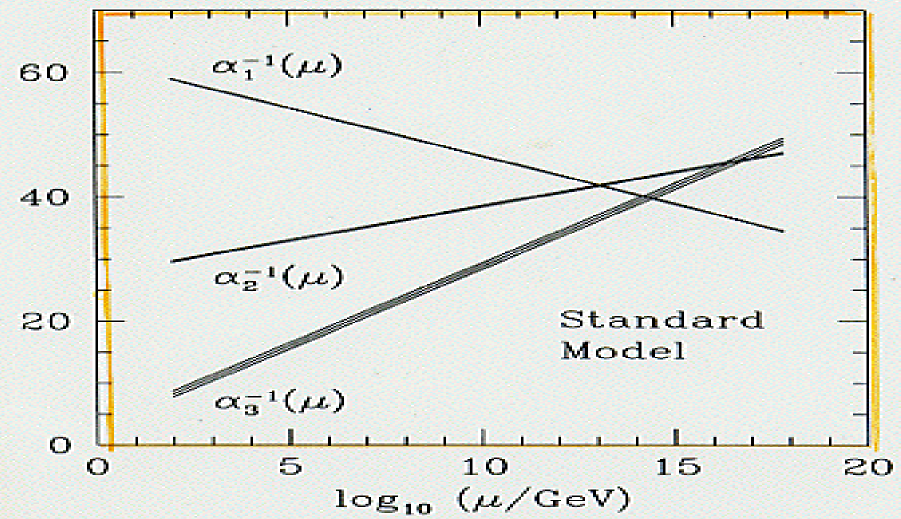


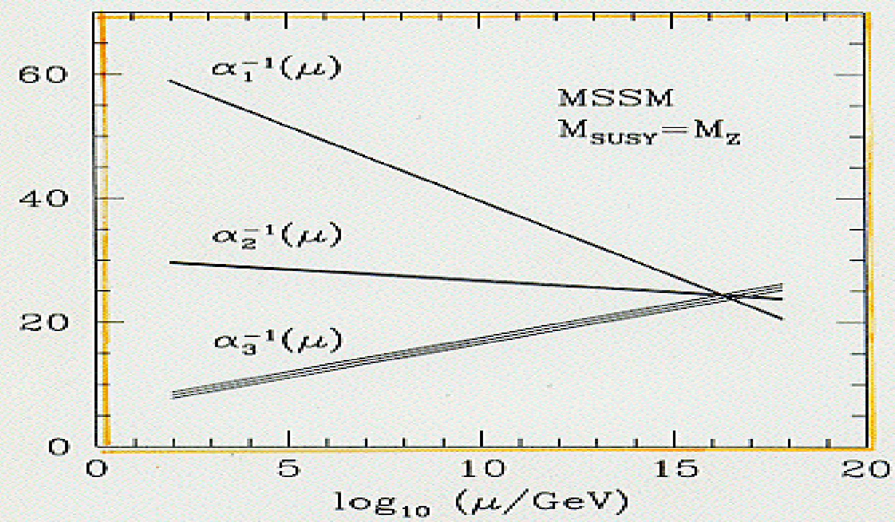
Average
Hadronic Jets
e^+e^- rates
e^+e^- event shapes
Fragmentation
Z width
ep event shapes
Polarized DIS
Deep Inelastic Scattering (DIS)
τ decays
Lattice
Υ decay
0.1
0.12
0.14
$\alpha_s(M_Z)$



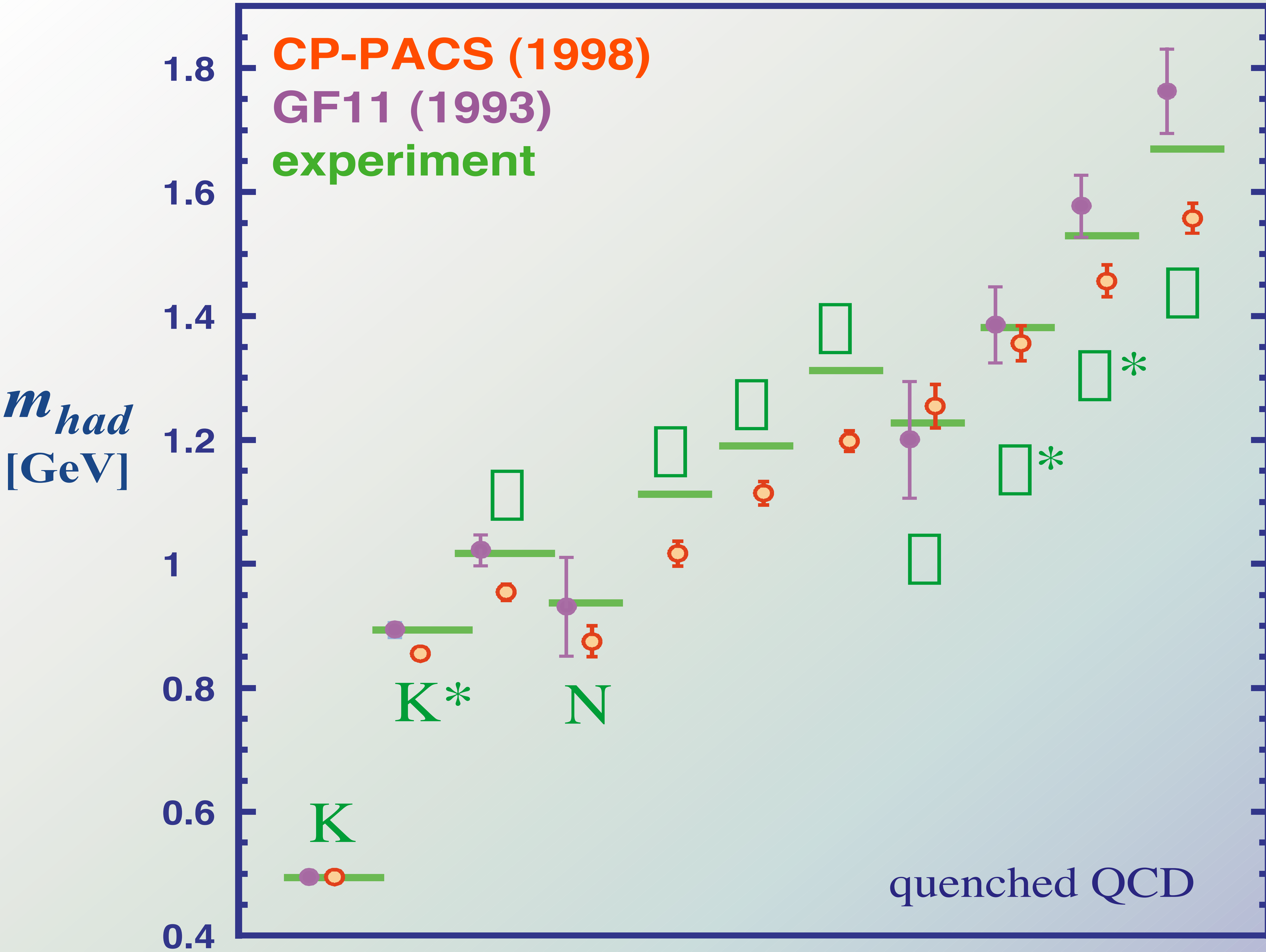
0.5
0.4
0.3
0.2
0.1
αs(Q)
Theory
Data
NLO
NNLO
Lattice
Deep Inelastic Scattering
e+e- Annihilation
Hadron Collisions
Heavy Quarkonia
Λ(5) MS
αs(MZ)
QCD
O(αs4)
251 MeV 0.1215
213 MeV 0.1184
178 MeV 0.1153
1
10
100
Q [GeV]

Unification of gauge couplings...



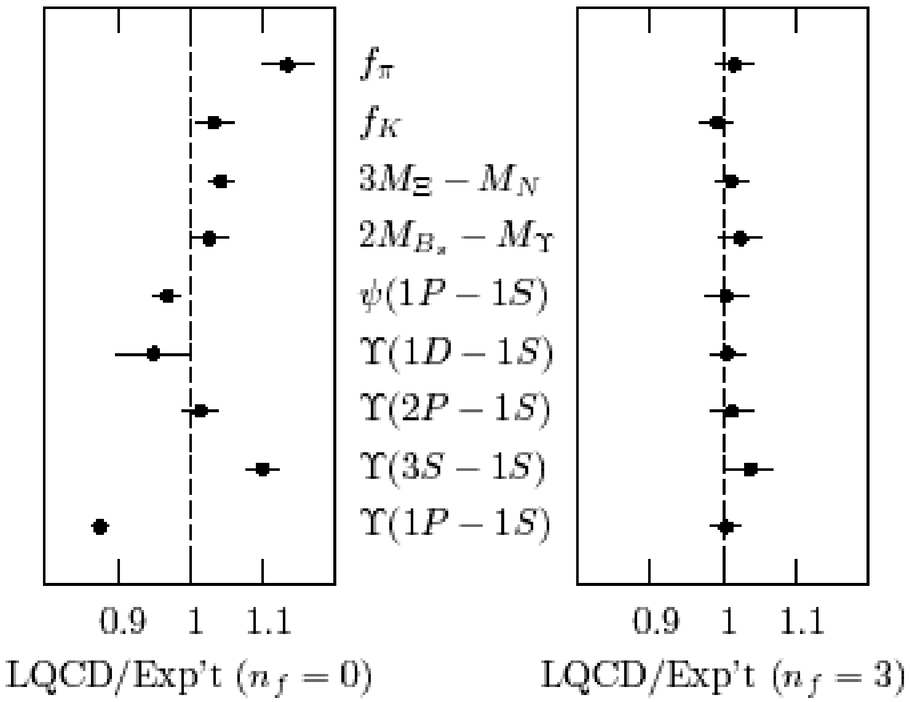


Masses and Matrix Elements



CP-PACS (1998)
GF11 (1993)
experiment
m_{had}
[GeV]
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
K
K*
N
quenched QCD

• Gold-Plated Observables Davies et al, hep-lat/0304004



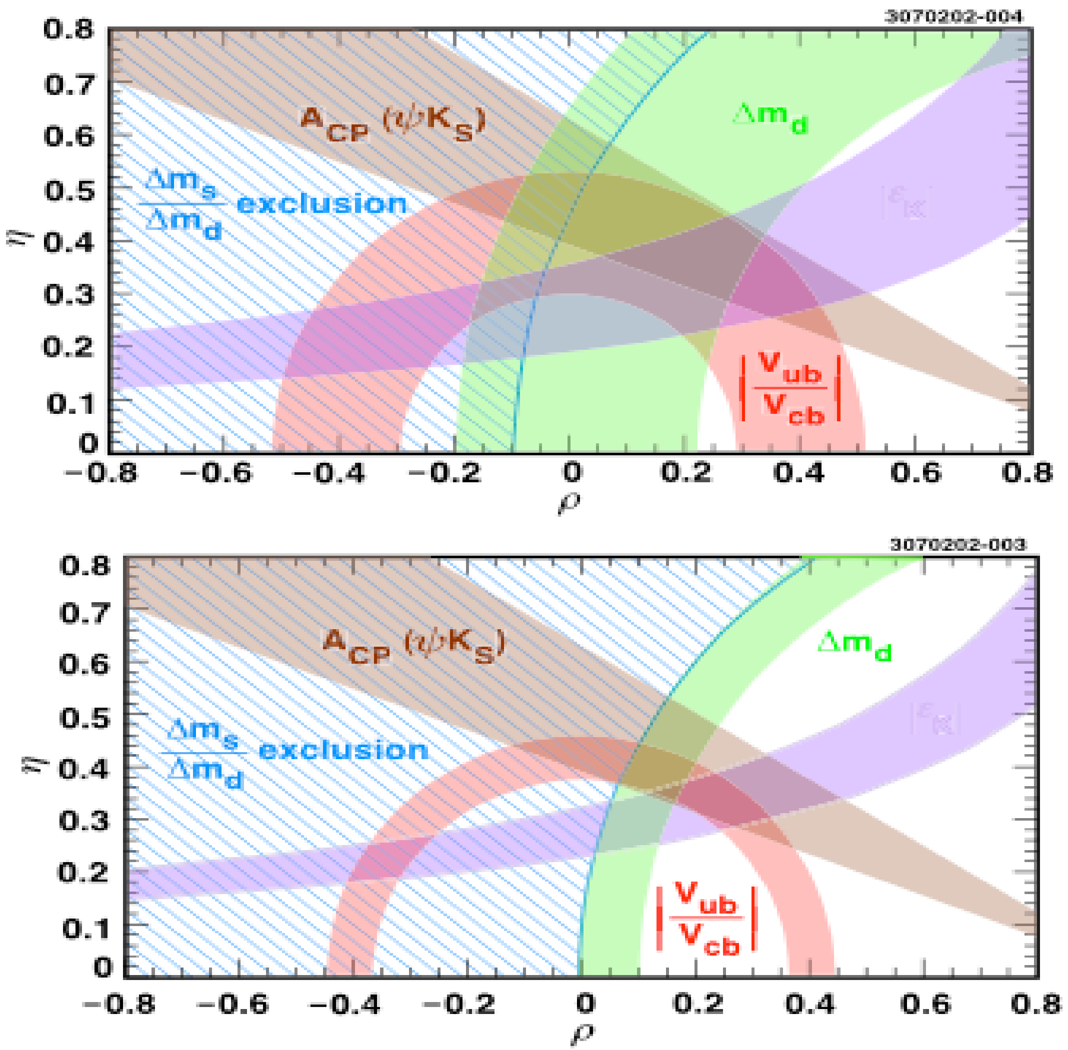
Staggered quarks

Asqtad improved action

a = 0.13, 0.09 fm

Errors $\sim$ 3%

Gold-plated processes for 8/9 CKM elements

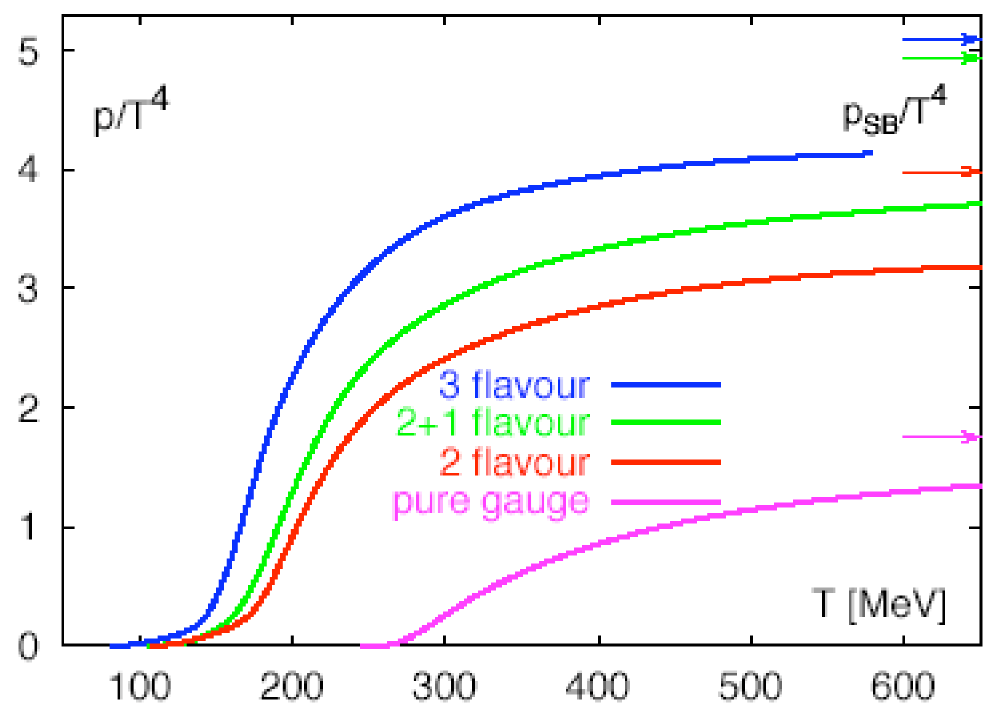


Constraints on the Standard Model parameters ρ and η (one sigma confidence level). For the Standard Model to be correct, they must be restricted to the region of overlap of the solidly colored bands. The figure on the top shows the constraints as they exist today. The figure on the bottom shows the constraints as they would exist with no improvement in the experimental errors, but with lattice gauge theory uncertainties reduced to 3%. R. Patterson, Cornell University.

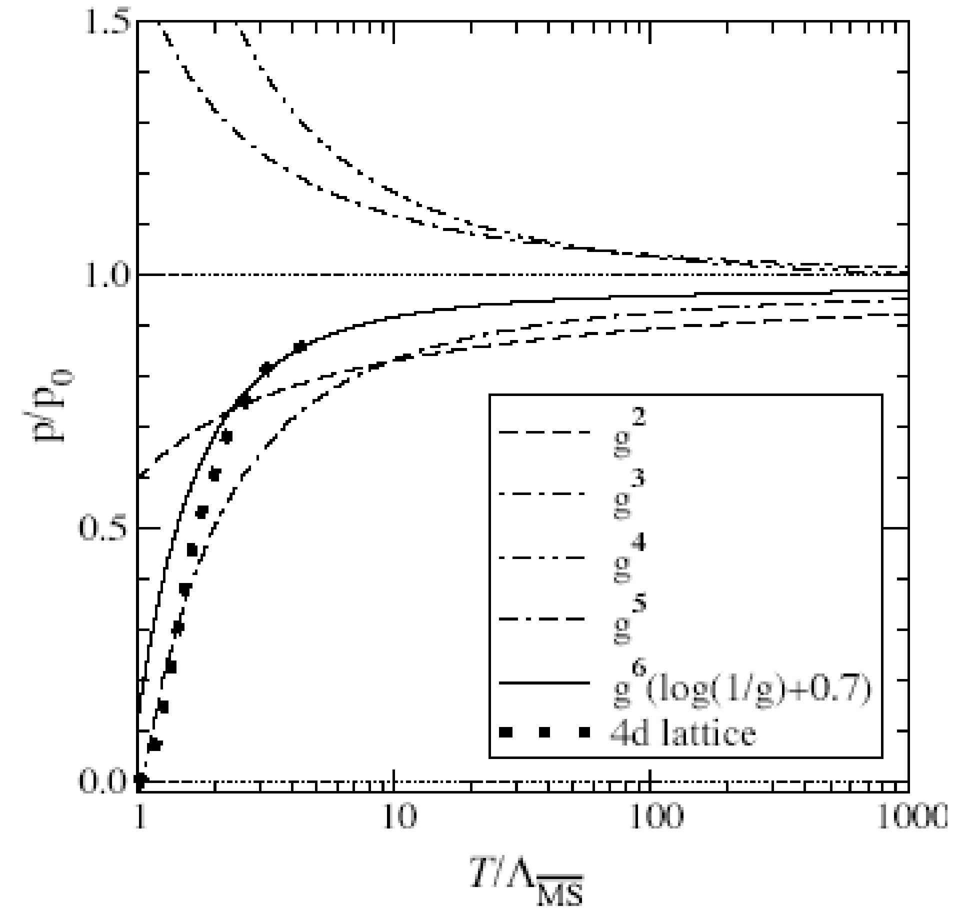
Extreme Conditions

Equation of State at $\mu = 0$

- **Equation of state** **(F. Karsch hep-lat/0106019)**



Matching to Perturbative QCD



(Kajantie, Laine, Rummukainen, Schroder hep-ph/0211321)

- Dimensionally reduced effective theory

  Integrate out 'hard' modes $\sim \pi T$

  Integrate out 'soft' modes $\sim gT$

  Calculate p_G in 3-D

  $p = 1 + g^2 + g^3 + g^4 + g^5 + g^6 \ln g + \# g^6$

Pentaquarks: A Case Study

Properties of the $\Theta^+(1540)$

★★ Mass 1540 MeV

★★ Width < 10 MeV

Some evidence that $<<$ 10 MeV! nucl-th/0308012

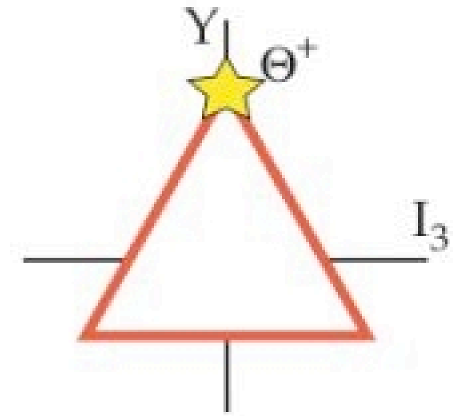
★★ K^+n quantum numbers:

- ⋆ $Y = 2$
- ⋆ $I_3 = 0$

★ $I = 0$ (no K^+p partner)

? $J = \frac{1}{2}$? Flat angular distribution

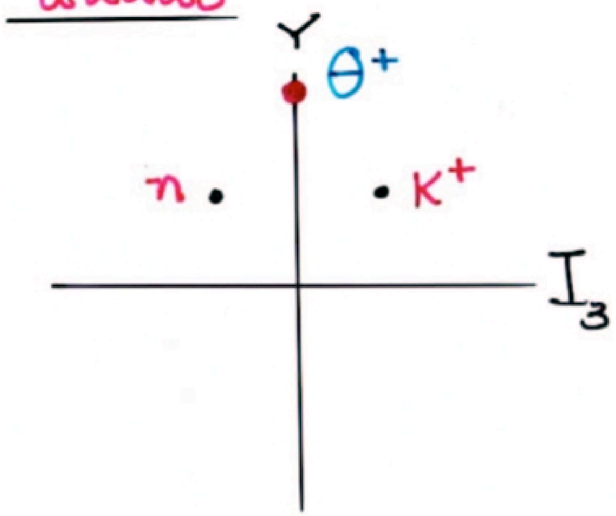
?? Parity unknown



- SPring-8 0301020
  DIANA(ITEP) 0304040
  CLAS 0307018
  Else 0307083, Hermes, . . .

BASIC FACTS ABOUT THE CHANNEL

* $K^+ n$ AT LEAST $uudd\bar{s}$



* $Y = 2$
  $I_3 = 0$

* TOTAL ISOSPIN

  - $I = 0$ ✓
  - $I = 1$ $[K^0 n]$ $[K^+ n + K^0 p]$ $[K^+ p]$

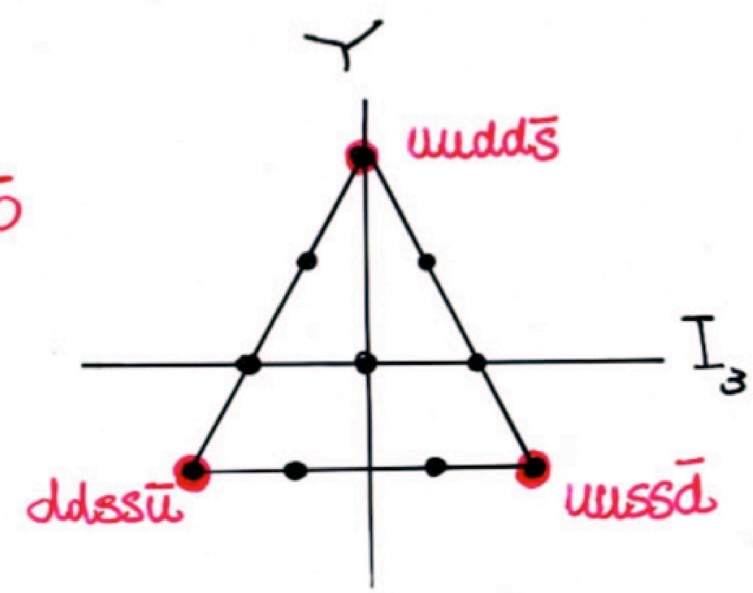
    NO EVIDENCE FOR $K^+ p$ PARTNER ?

  - $I = 2$! Capstick, Page, Roberts hep-ph 0307019

  ASSUME $I = 0$

* SU(3)-FLAVOR $\overline{10}$

  MORE LATER



Interpretations of the $\Theta^+(1540)$

- Chiral Soliton Model (motivated experiments)

  A narrow K^+n resonance generated by chiral dynamics.

  Chemtob (1984) ; Praszalowicz (1987)
  Diakonov, Petrov, Polyakov (1997)

- Uncorrelated Quark Model

  $Q^4\bar{Q}$ in the lowest orbital of some mean field: NRQM, bag, ...

  RLJ (1976), Strottman (1978), Wybourne
  Carlson, Carone, Kwee, & Nazaryan,...

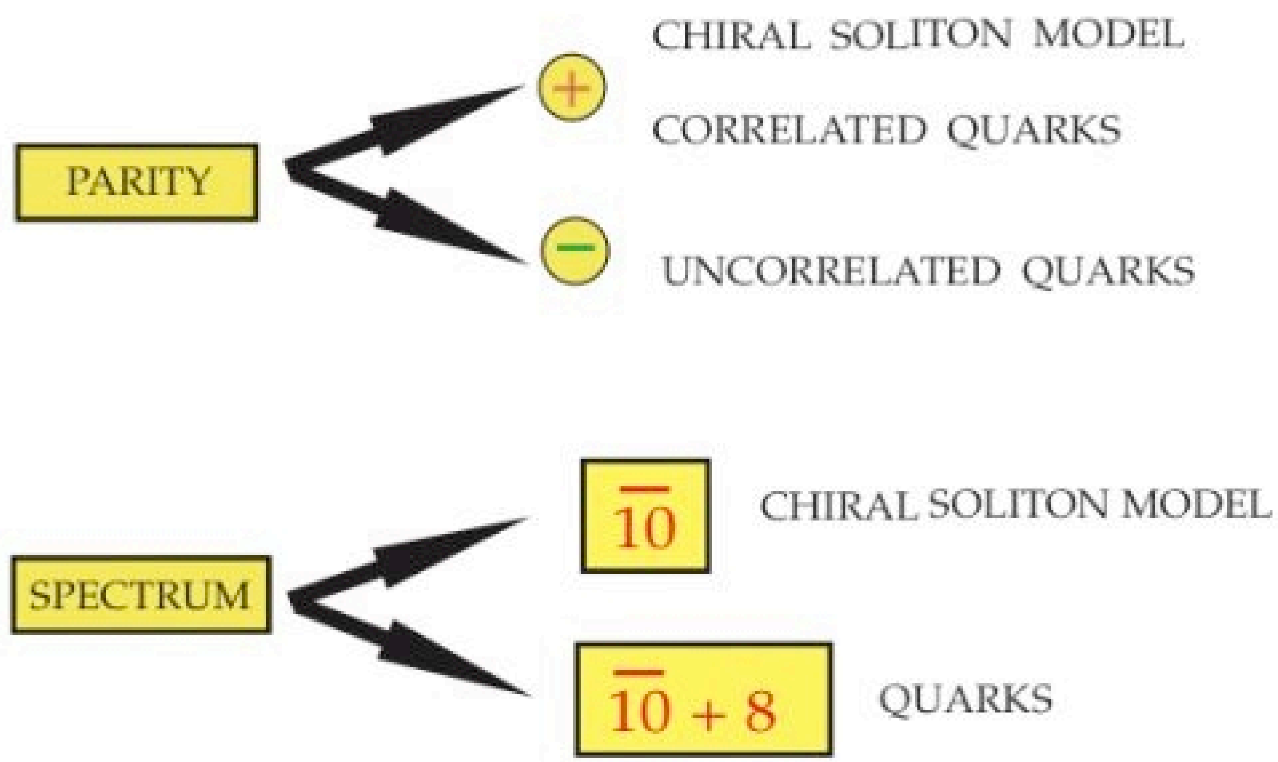
- Correlated (Diquark) Description

  $[QQ]$ correlated in an antisymmetric color, flavor, and spin state.

  Jaffe & Wilczek

Essential Distinctions

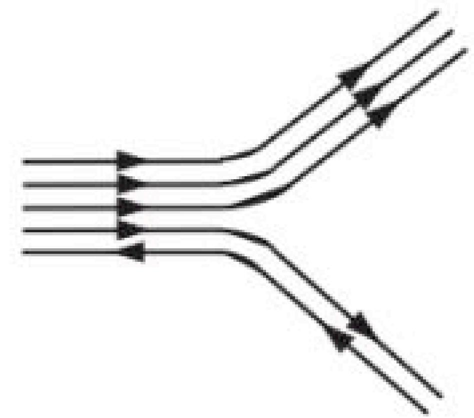
These three pictures make strikingly different predictions for the parity of the $\Theta^+(1540)$ and the spectrum of particles in the same class as the $\Theta^+(1540)$



Thoughts on the $\Theta^+(1540)$ as a K^+n Resonance

⋆ At $M_\Theta = 1540$ MeV kaon and nucleon are

$\approx$ non-relativistic



⋆ KN is only channel until $K\Delta$ opens at 1725 MeV.

⋆ No quark annihilation diagrams!

So – if there is no dynamics that is hidden from the KN sector

It must be possible to describe the $\Theta^+(1540)$ in terms of non-relativistic KN potential theory!

- Potentials do not generate resonances in the s-wave: bound states, virtual states, smooth phases...

- Try p-wave: What potential generates a resonance at p =270 MeV and Γ <10 MeV?

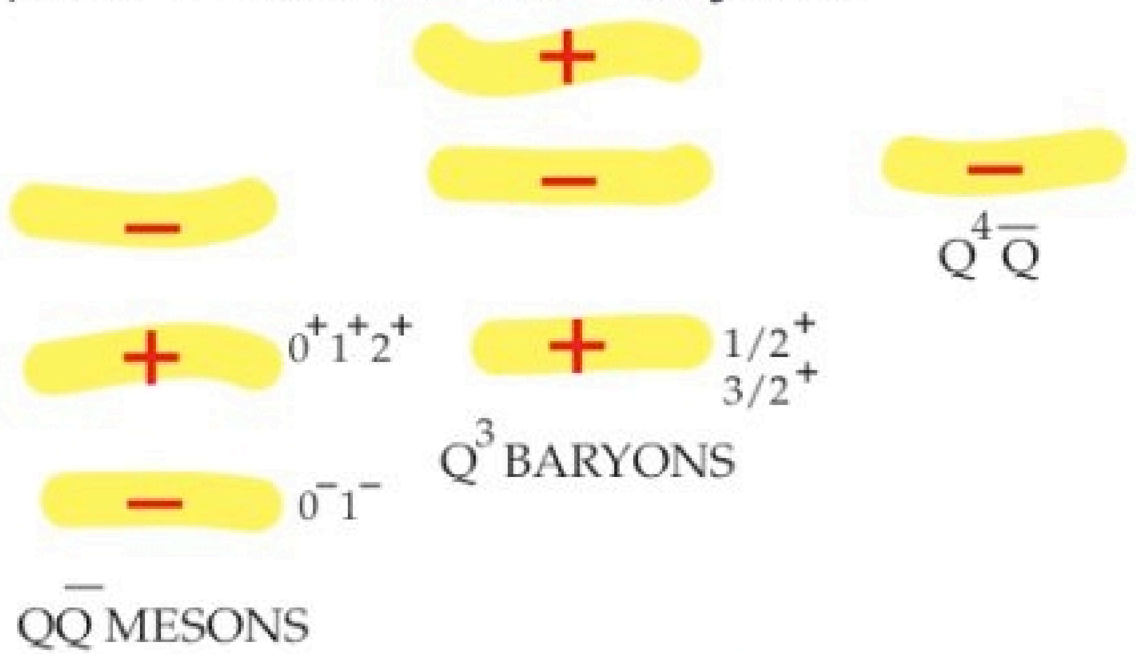
- $pR \approx 1.5 \Rightarrow \Gamma \approx 200$MeV!

- {Range & Depth}⇔{Mass & Width}

Part III: Dynamics

I. Uncorrelated Quarks

Quarks in a mean field give good description of first two supermultiplets of mesons and baryons.



And parity of lowest multiplet of $Q^4\overline{Q}$ is negative (Q and $\overline{Q}$ have opposite intrinsiic parity).

Comments on the uncorrelated quark model

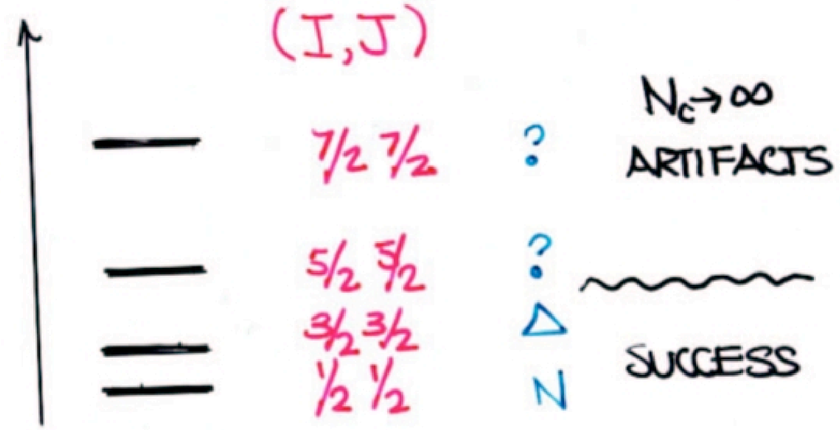
- Hundreds (1260 to be precise!) of negative parity baryons around the $\Theta^+(1540)$. Most of which are lost in the continuum, but some cannot be promoted to high energy (see below).
- Including $uudd\bar{s} \Rightarrow uudd\bar{d}$ & $uudd\bar{u}$. A negative parity "nucleon" lighter than the $\Theta^+(1540)$ for which there is no evidence.
- Negative parity. So the $\Theta^+(1540)$ and other $Q^4\overline{Q}$ lie in the KN s-wave, where their widths are problematic.

SKYRME (CHIRAL SOLITON) MODEL

- $N_c \to \infty$ (BARYONS AS SOLITONS)
- CHIRAL DYNAMICS (PIONS!)

→ TWO FLAVOR

LEARNED HOW TO CHOOSE WHAT WORKS



→ THREE FLAVOR

EARLY WORKERS DISCARDED TO WITH OTHER $N_c \to \infty$ ARTIFACTS

→ BUT SOME TOOK IT SERIOUSLY

- CHEMTOB 1983-4
- PRASZALOWICZ 1987: $M(\Theta^+) = 1530$ MeV !
- DIAKONOV, PETROV, POLYAKOV 1997

$M(\Theta^+) = 1530 \quad \Gamma(\Theta^+) < 15$

INFLUENCED EXPERIMENTERS

Reasons for Skyrme-Reserve

Skyrme phenomenology has a mixed record.

Its connection to QCD fundamentals is somewhat tortuous.

It leads to a counterintuitive spectrum, as we'll see.

Diquarks

In QCD, the color antitriplet channel is very attractive. This might introduce important correlations that are not incorporated in the traditional quark model.

As a crude way to implement this dynamics, consider that quark pairs can form color antitriplet, flavor antitriplet spin-0 bosons with short-range repulsion.

Diquarks – Some Evidence

Notation: $QQ\ \bar{3}_c\bar{3}_f\mathbf{0}$: $[ud]$ $[ds]$ $[su]$

- Color superconductivity

  $[QQ]^{\bar{3}_c}{}_{\bar{3}_f}$ condenses at high energy:

  $$\langle [QQ]^{\alpha}{}_{a} = v\delta^{\alpha}{}_{a}$$

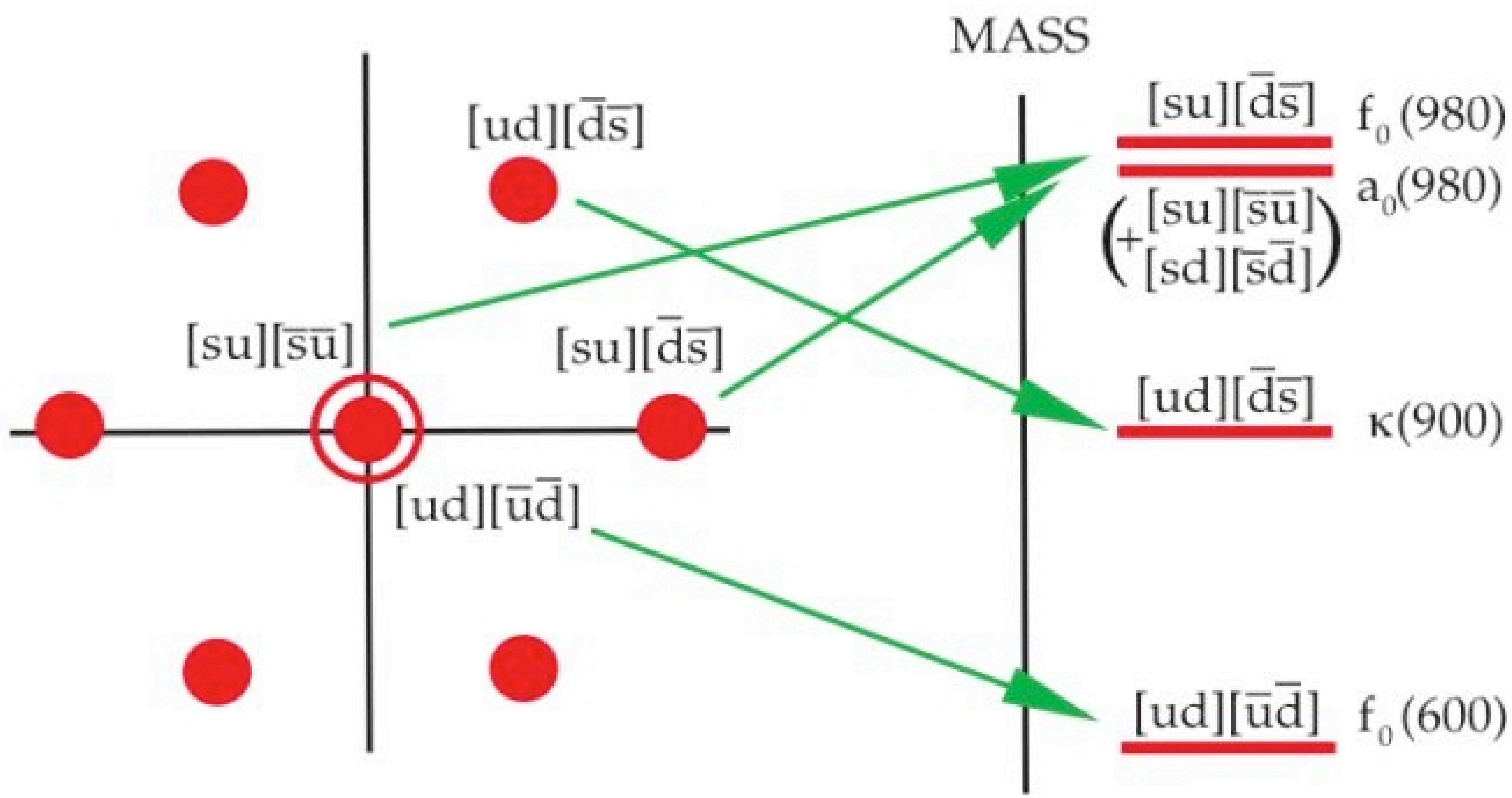
  Proven at high density, phenomenology in quark matter. . .

- Scalar mesons

	Q^2		$\bar{Q}^2$	$Q^2\bar{Q}^2$
COLOR	$\bar{3}$	$\otimes$	3	1
FLAVOR	$\bar{3}$	$\otimes$	3	$1 \oplus 8$
SPIN	0	$\otimes$	0	0

- $\Rightarrow$ A nonet of 0^{++} mesons

One Slide Summary of Scalar Mesons



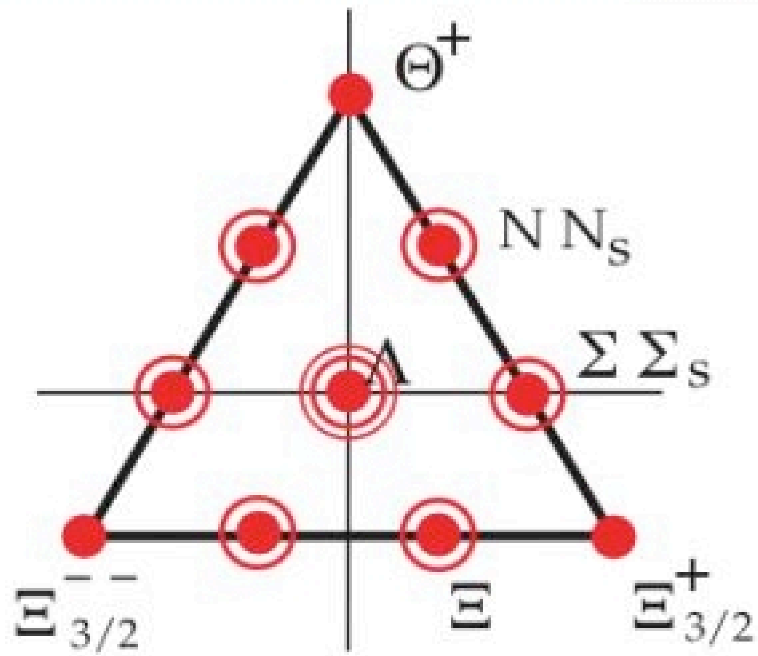
Diquarks and $Q^4\bar{Q}$ $[Q_1Q_2]$ is a boson

- Diquark — diquark wavefunction: two color $\bar{3}_C$ bosons: $[Q_1Q_2]^{\bar{3}_C} \otimes [Q_3Q_4]^{\bar{3}_C}$ must couple to 3_C to join antiquark in a color singlet hadron.

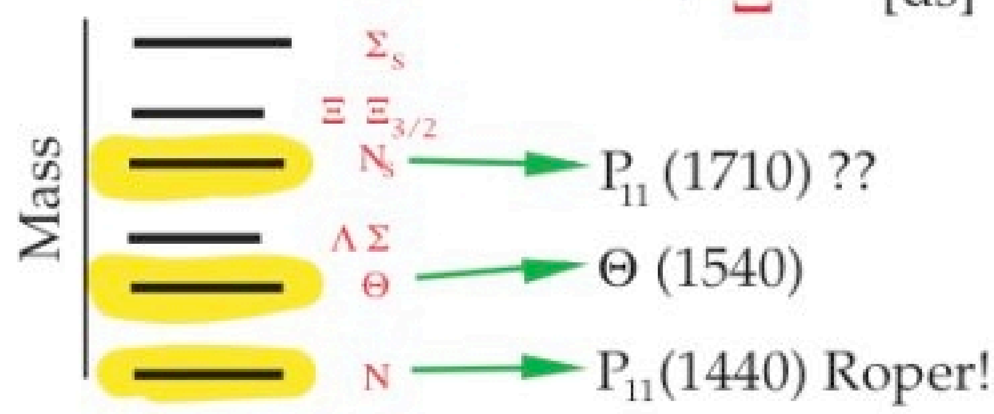
$$\left[\left[(3_C \otimes 3_C)^{\bar{3}_C} \otimes (3_C \otimes 3_C)^{\bar{3}_C}\right]^{3_C} \otimes \bar{3}_C\right]^{1_C}$$

- Consider identical diquarks as in the Θ $[ud] - [ud]$ The overall diquark-diquark wavefunction must be exchange symmetric (bosons). It is antisymmetric in color. So it must be antisymmetric in space!! $\Rightarrow$ <u>ODD PARITY</u>

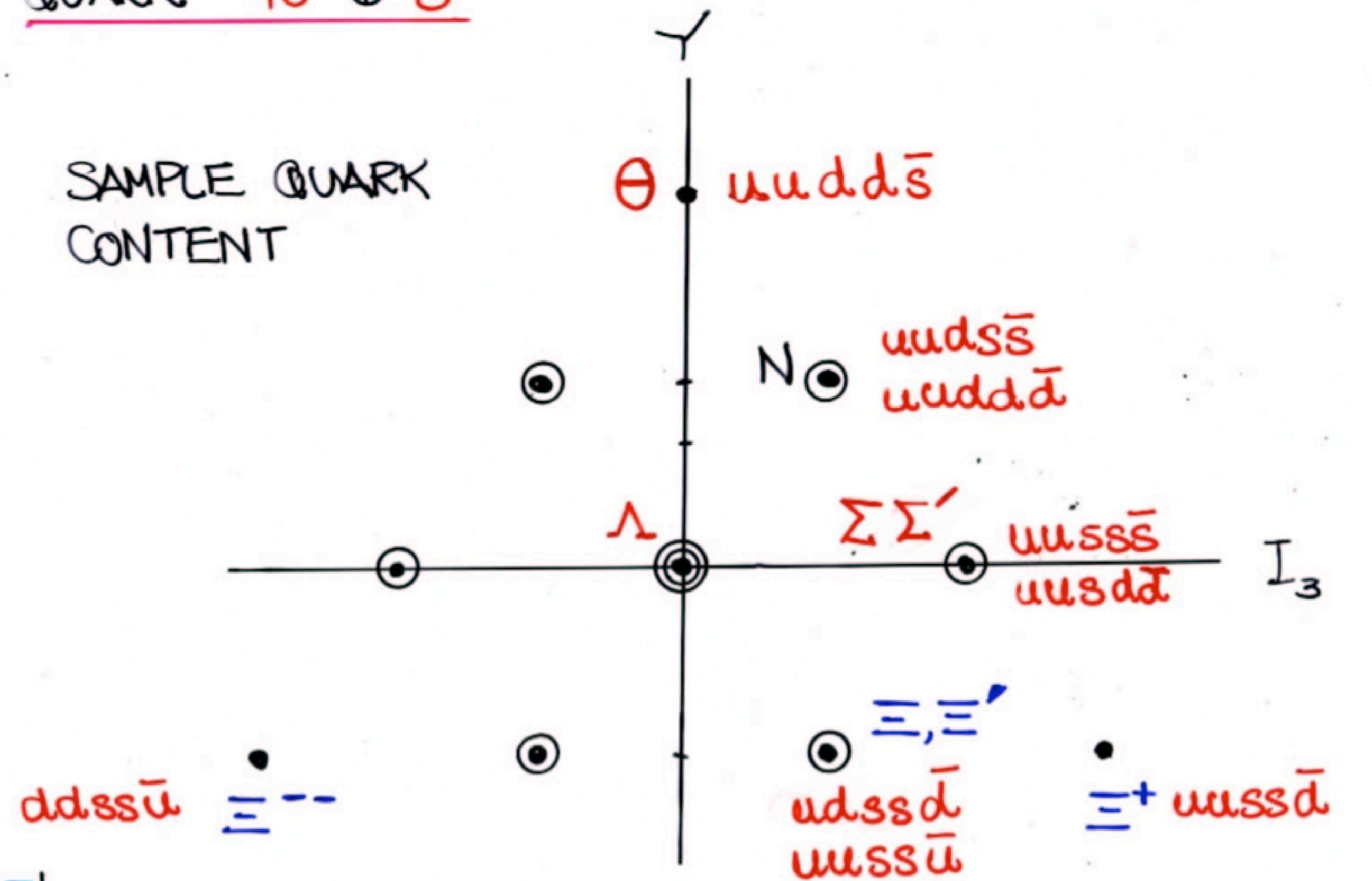
$[QQ]^2\bar{Q}$ Octet and Antidecuplet

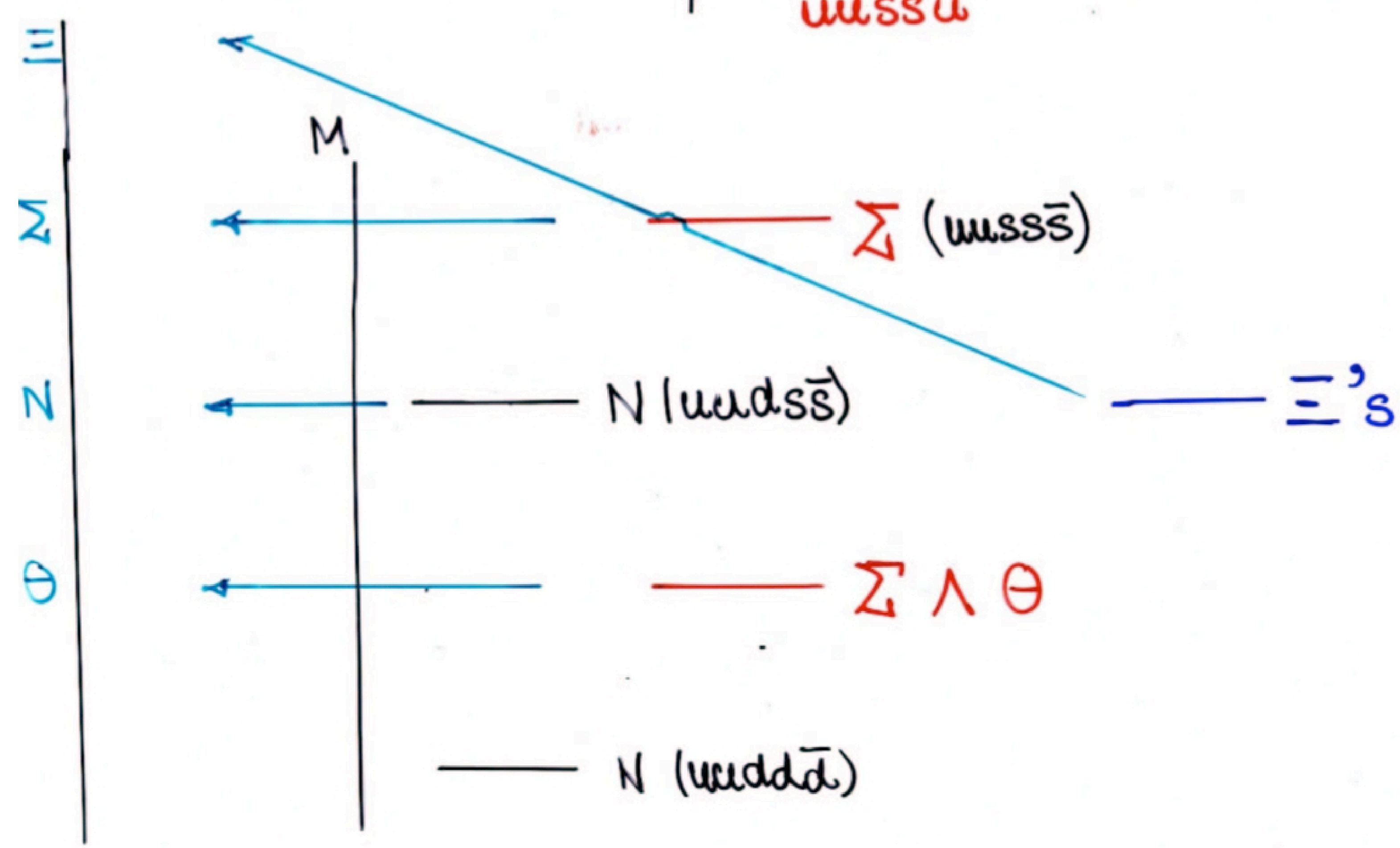


➡ Θ^+	$[ud]^2\bar{s}$
➡ N	$[ud]^2\bar{d}$
➡ N_S	$[ud][su]\,\bar{s}$
$\Lambda\,\Sigma$	$[ud][su]\,\bar{d}$, ...
Σ	$[us]^2\bar{s}$
Ξ	$[su][ds]\,\bar{d}$, ...
➡ Ξ^{--}	$[su]^2\bar{d}$
➡ Ξ^+	$[ds]^2\bar{u}$



QUARK $\overline{10} \oplus 8$





Identifications, Postdictions and Predictions

- Simple Hamiltonian for $SU(3)$violation:

$$M(n_s, n_{\bar{s}}) = M_0 + \alpha n_s + (n_s + n_{\bar{s}})\Delta_s$$

  where Δ_s is the mass effect and $\alpha = \frac{3}{4}(M_\Sigma - M_\Lambda) \approx 60$ MeV, is the cost of $[ud] \to [us]$ beyond the mass effect.
- Non-strange state lighter than $\Theta^+(1540)$ can be identified with the $P_{11}(1440)$– the "Roper".
  - ⋆ Roper has been problematic for 40 years because of its positive parity.
  - ⋆ Roper and $\Theta^+(1540)$ fix the parameters in a simple $SU(3)$ violation Hamiltonian.
- N_s comes out near existing $P_{11}(1710)$ and exotic Ξ states are light!

Widths

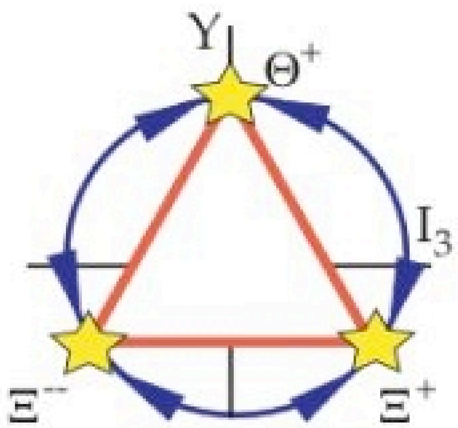
- Mechanism for reduction of widths below KN potential theory estimates: Resonance configuration may have small overlap with KN:

$$\left[[ud]^2\,\bar{s}\right]_\Theta \overset{?}{\Leftrightarrow} [udd]_n\,[u\bar{s}]_K$$

- Relating $\Gamma(\Xi^{--})$ to $\Gamma(\Theta^+)$

$$\begin{aligned}\Theta &\to K^+ n/K^0 p\\ \Xi^{--} &\to \Xi^-\pi^-/\Sigma^- K^-\end{aligned}$$

$SU(3)_f$ for matrix elements combined with p-wave phase space.



Predict $\Gamma(\Xi^{--}(1750)) \approx 1.4\Gamma(\Theta)$ SMALL!.

Searching for Exotic Cascades

- Robust prediction: Degenerate with $\Theta^+(1540)$ in the $SU(3)_{\mathsf{f}}$ limit.
- $SU(3)_{\mathsf{f}}$ symmetry breaking? $\bar{s} \to ss \Leftrightarrow \approx$ 200 MeV.
- Sample reactions:

$$K^-\, d \to K^+\, p\, \Xi^{--}$$

$$\begin{aligned} \Xi^{--} &\to \Xi^-\, \pi^- \\ \Xi^- &\to \Lambda^0 \pi^- \\ \Lambda^0 &\to p\, \pi^- \end{aligned}$$

$$K^-\, d \to K^+\, p\, p\, \pi^-\, \pi^-\, \pi^-$$

$$\gamma\, d \to K^+\, K^+\, p\, \Xi^{--}$$

Production of Ξ^+ is more difficult because of ambiguous strangeness of neutral kaons.

- Exciting prospect for JLAB, JHF, and AGS at Brookhaven?

We estimate that the (ud)(ud)c-bar and (ud)(ud)b-bar pentaquarks are **stable** against strong decays!

Diquarks Everywhere?

Color superconductivity

Scalar nonet phenomenology

Roper

No H

$\Delta I = 1/2$ rule

Theoretical Approaches

Quarks→diquarks using extra "fictitious" gauge interactions

Lattice simulations, especially of pentaquarks with heavy spectators

Creative Opportunities

We can consider different numbers and mass spectra of quarks. E.g., with four flavors the question of “quadriquark” exotics gets sharp.

By tailoring sources, we can explore wave functions (form factors, structure functions). E.g., we can probe for diquark correlations.

Using these tools, we can test and refine dynamical models and approximation schemes.